# Math 55 Quiz 2 DIS 106

Name: ______________________________ 7 Feb 2022

1. Show that there exists a unique positive integer $x$ such that $x^2 + 6x = 16$.

   Notice that $2^2 + 6 \times 2 = 16$, hence $x^2 + 6x = 16$ has a solution for positive integer $x$. For uniqueness, suppose that $x^2 + 6x = 16$ for some positive integer $x$, then

   $$(x-2)(x+8) = x^2 + 6x - 16 = 0$$

   Hence $x - 2 = 0$ or $x + 8 = 0$; in other words $x = 2$ or $-8$. $x$ is positive hence is not $-8$, so $x = 2$. This shows that 2 is the unique positive integer solution for $x$.

2. Prove or disprove that for all sets $A, B$,

   (a) $A \cap B \subseteq A \cup B$

   (b) $A \cap (\overline{A} \cup B) = B$

   (a) This is true. Suppose $x \in A \cap B$. This means that $x \in A$ and $x \in B$. Hence $x \in A$ or $x \in B$, so $x \in A \cup B$.

   (b) This is false. Suppose $U = \{1, 2, 3\}, A = \{1, 2\}, B = \{2, 3\}$. Then $A \cap (\overline{A} \cup B) = \{2\} \neq \{2, 3\} = B$